MATEMÁTICOS ACTUALES

Palle Erik Tikob Jorgensen, Álgebra de operadores, análisis armónico, física aplicada

Palle Jorgensen era hijo de Soren A W Jorgensen y Gyrit D Baden. El padre de Palle, Soren Jorgensen, era ingeniero eléctrico, mientras que su madre Gyrit era profesora de matemáticas. Palle Jorgensen escribe, Referencia [5]:

> *Mi abuelo materno, Rasmus Bortmann Tikøb Baden, era un erudito bíblico y estaba profundamente interesado en la ciencia. Una cita del pastor Baden que se me queda grabado en la mente (debo haber tenido 4 o 5 años en ese momento) es esta: "Las matemáticas y la física son la sabiduría de Dios, mientras que las ciencias sociales representan el estudio de la estupidez de la humanidad." A esta edad tan temprana, los tres, mi madre, mi padre y el pastor Baden, influyeron profundamente en mi pensamiento. Siempre creo que lo que aprendemos en los primeros 5 a 10 años de nuestra vida tiene un impacto mucho mayor que lo que viene después. Las primeras influencias de mi madre incluyen la geometría, los números complejos y la fórmula de Taylor. Mi madre era muy buena en idiomas. Quizás nuestro aprendizaje de las matemáticas tenga mucho en común con el de los idiomas. En retrospectiva, me di cuenta de que adquirí una facilidad con tres idiomas "extranjeros" de mi madre; mucho tiempo atrás en mis años de formación. Al mismo tiempo, al escuchar a mi padre hablar sobre los filtros de paso alto y paso bajo y la transmisión de larga distancia, aprendí los fundamentos del procesamiento de señales. Recuerdo esta inspiración para el procesamiento de señales en mi investigación mucho más tardía sobre multiresoluciones de ondículas; y lo menciono en mi libro Wavelet de 2002.*

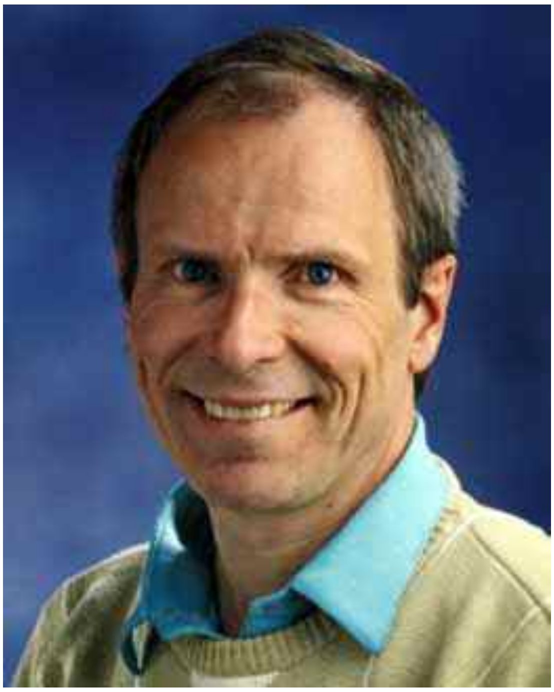
Entró en un poco más de detalle sobre la influencia del trabajo de su padre en los filtros de paso alto y paso bajo en [4] (ver también Referencia [2]):

Cuando era pequeño, crecí en Dinamarca, antes del jardín de infancia; solo que no había ninguno entonces ..., - así que antes de oír hablar de 'The Tinder Box' o 'The Ugly Duckling' de los libros de Hans Christian Andersen, mi padre me habló de los filtros de paso bajo y paso alto . Era ingeniero telefónico y trabajó en los filtros utilizados en las señales transmitidas a través de cables largos, justo después de la guerra, la Segunda Guerra Mundial. La parte 'alta' y 'baja' de la historia se refiere a las bandas de frecuencia de las señales de sonido. No es que esto signifique mucho para mí en ese momento. Más bien, estaba fascinado con las imágenes en los diarios de EE que estaban apiladas en el piso junto a mí, y pasé horas mirándolas [-¡eso era todo lo que había, allí en el piso!], Así que estas imágenes de filtro el diseño, algunos en color, me ocupaba los domingos largos mientras mi padre construía instrumentos en la sala de estar. ¡No tengo nada más que hacer! Luego, después de ir a la escuela, me olvidé por completo de la explicación de mi padre sobre los filtros de espejo en cuadratura; no es de extrañar (!!), y estuvieron fuera de sí durante mucho tiempo. Nunca tuve ninguna razón en particular para pensar mucho en ellos, me refiero a las bandas de frecuencia de paso bajo y todo eso, pero estoy seguro de que de alguna manera extraña crearon una impresión visual duradera para mí.

Escribió sobre las influencias posteriores sobre él en Referencia [5]:

Cuando era niño, de los 11 a los 17 años, en mis vacaciones escolares de verano, hice viajes por mi cuenta, haciendo autostop, a campos de trabajo para jóvenes en el extranjero, en Inglaterra, Alemania, Francia y (un grupo de jóvenes sionistas) en Israel. , en los kibutzim. Más tarde en la escuela secundaria y en la universidad, tuve la suerte de haber tenido profesores muy inspiradores; pero especialmente afortunado de haber sido parte de una generación de estudiantes con ambiciones de sobresalir en matemáticas y ciencias. El paralelo más cercano a la escuela secundaria en Dinamarca se llama Gymnasium. El nombre mío era Marselisborg Gymnasium en Aarhus. Tutorial de Dinamarca: este fue un programa de 3 años, y tuve la suerte de contar con grandes profesores en matemáticas, ciencias e idiomas. El plan de estudios en ese momento incluía cálculo y álgebra lineal. (El plan de estudios probablemente se ha diluido desde entonces). Otro de mis intereses de esta época incluye la economía.

Jorgensen ingresó en la Universidad de Aarhus y obtuvo un B.A. en 1968, un M.S. en 1970 y un Ph.D. en Matemáticas en 1973 por su tesis Representaciones de dimensión infinita de álgebras de Lie y grupos de Lie. Hubo muchas influencias importantes en Jorgensen durante estos años. Uno fue Kiyosi Ito quien, aunque fue profesor en la Universidad de Kyoto desde 1952 hasta que se jubiló en 1979, ocupó



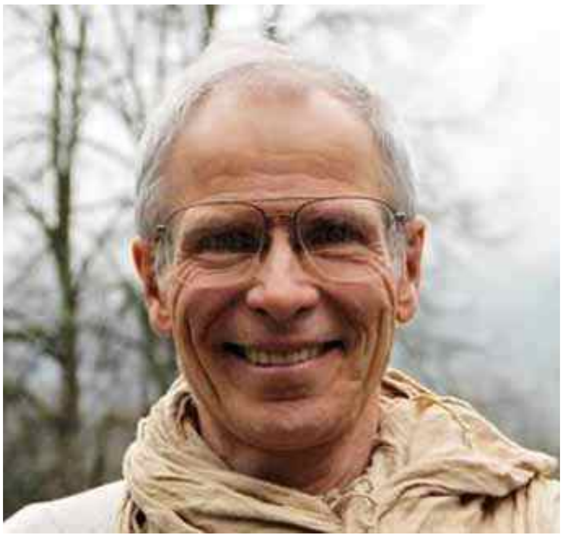
un puesto como profesor en la Universidad de Aarhus en los años 1966-69 mientras Jorgensen era un estudiante de posgrado. Su asesor de tesis en Aarhus fue Niels Kristian Skovhus Poulsen, quien se había graduado con un doctorado. en 1970 asesorado por Irving Segal. Jorgensen escribe Referencia [5]:

En la Universidad de Aarhus, tuve excelentes profesores de matemáticas, y de mis compañeros estudiantes de posgrado en matemáticas, más de un puñado terminaron como matemáticos profesionales y en las mejores universidades, tanto en Dinamarca como en los Estados Unidos. Uno de mis primeros y especialmente inspiradores profesores de matemáticas, y de mi generación, que se destaca es Svend Bundgaard. De hecho, el profesor Svend Bundgaard todavía existía a principios de la década de 1980 cuando yo estaba en la facultad del Departamento de Matemáticas de la Universidad de Aarhus.

Digamos un poco más sobre Svend Bundgaard (1912-1984). Fue educado en la Universidad de Copenhague y, después de enseñar en esa universidad, fue nombrado presidente de la recientemente establecida Facultad de Ciencias de la Universidad de Aarhus en 1954. Desempeñó un papel importante en la expansión de esa facultad, en particular el Instituto de Matemáticas. , sirviendo como director de la universidad en los años 1971-1976. También fue cofundador de la revista Mathematica Scandinavica.

Los resultados de la tesis de Jorgensen pasaron a formar parte de la posterior monografía Operador de relaciones de conmutación. Relaciones de conmutación para operadores, semigrupos y solventes con aplicaciones a la física matemática y representaciones de grupos de Lie escritas conjuntamente con Robert T Moore y publicadas en 1984. Comienza una revisión de Derek W Robinson Referencia [13]:

En los últimos treinta años ha florecido la teoría de los grupos continuos de un parámetro y semigrupos en los espacios de Banach. Esta teoría, que tiene sus raíces históricas en la descripción dinámica de los sistemas mecánicos cuánticos, ha encontrado posteriormente aplicaciones en muchos campos de las matemáticas. El énfasis principal de este desarrollo ha estado en el análisis de grupos individuales o semigrupos y sus generadores. Se ha dedicado menos esfuerzo al estudio de la estructura analítica de familias de grupos no trabajadores y semigrupos. Una vez más, estos problemas encontraron su motivación más temprana en la teoría cuántica, en particular con la formulación de Heisenberg en términos de familias de observables no conmutados. Problemas de esta naturaleza constituyen el objeto principal de la monografía en revisión y los autores intentan, con cierto éxito, mostrar que el análisis de las relaciones de conmutación de operadores conduce a una unificación de diversas áreas de las matemáticas.



Después de la obtención de su doctorado, Jorgensen recibió una beca postdoctoral de varios años del Danish Natural Science Research Council que le permitió realizar investigaciones en universidades en el extranjero; Se alentaron encarecidamente los viajes al extranjero y los contactos internacionales. Se fue a los Estados Unidos en 1973 y, con el apoyo de la beca, se convirtió en académico invitado en la Universidad de Princeton. En 1975 publicó su primer artículo *Representaciones de operadores diferenciales en un grupo de Lie* que trataba sobre procesos de Hunt en grupos de Lie. Escribe, Referencia [5]:

Esta investigación se inspiró en gran medida en la investigación pionera de Ed Nelson y Lars Hörmander.

La dirección en el artículo es Departamento de Matemáticas, Universidad de Princeton, y la investigación fue patrocinada por la Universidad de Odense, Dinamarca. La forma en que estas becas postdoctorales funcionaban en Dinamarca en este momento significaba que la financiación procedía del Consejo Danés de Investigación en Ciencias Naturales, pero la investigación fue patrocinada por una o más de las universidades. En el caso de Jorgensen, fue patrocinado por la

Universidad de Odense y la Universidad de Aarhus, Dinamarca. El trabajo tiene los siguientes reconocimientos:

El autor está en deuda con un gran número de matemáticos por sus útiles discusiones. Aprendió sobre las representaciones grupales de Niels Skovhus Poulsen. El profesor M Reed originalmente mostró interés en la idea de observar expresiones cuadráticas de generadores infinitesimales. Los profesores V Bargmann, G B Folland, R Goodman, A Klein, R T Moore, E Nelson, E Stein y A S Wightman han mostrado interés en varios resultados parciales en el camino. Los profesores J J Kohn y F Treves han sido útiles con ciertos aspectos del operador diferencial. El árbitro ha sugerido valiosas mejoras. Finalmente, el autor desea agradecer a Gregg Zuckerman por muchas discusiones útiles sobre las representaciones de los grupos de Lie.

El siguiente artículo de Jorgensen fue *Reducción aproximada de los subespacios para operadores lineales ilimitados* (1976) y, en ese momento, se había trasladado a la Universidad de Pensilvania en Filadelfia. Escribe en los Agradecimientos:

Es un placer agradecer al profesor S Sakai por sugerir el problema y por su constante aliento.

En Referencia [5] Jorgensen explica que su investigación durante su tiempo en la Universidad de Pensilvania se movió en la dirección de álgebras de operadores y física matemática. Se inspiró en Richard V Kadison, su asesor postdoctoral, en Robert T Powers y en Shôichirô Sakai.

Jorgensen se casó con Soon-Min Park el 4 de enero de 1975. Tuvieron tres hijos, Anton Yang Jorgensen (nacido el 24 de octubre de 1975), Greta Soon Jorgensen (nacida el 6 de septiembre de 1978) y Tina Soon Jorgensen.

En el verano de 1976, Jorgensen fue miembro invitado del Instituto de Investigación de Verano de la NSF en Teoría del Operador en la Universidad de New Hampshire. Después de trabajar en la Universidad de Pensilvania hasta 1977, se trasladó a la Universidad de Stanford en ese año, donde se convirtió en profesor asistente, Referencia [5]:

Las influencias importantes de finales de la década de 1970 incluyen a Ralph S Phillips, Hans Samelson, Paul Cohen, Kai Lai Chung, James McGregor y ... en Stanford. Jorgensen fue profesor asistente en la Universidad de Stanford en ese período, y su investigación se inspiró especialmente en la facultad de Stanford y los estudiantes graduados, por ejemplo, Peter Sarnak, en ese momento.

En 1979, Jorgensen solicitó la naturalización en el tribunal de distrito de San Francisco. Fue apoyado por dos testigos, Soon Min Jorgensen, su esposa, y William Arveson (1934-2011), quien fue profesor de Matemáticas en la Universidad de California, Berkeley, investigando sobre álgebras de operadores. En ese momento Jorgensen vivía en California Street, Mountain View, Santa Clara, California. Se le otorgó un certificado de naturalización el 24 de julio de 1979. Se le otorgó una beca del Consejo de Investigación Danés y fue al Instituto de Matemáticas de la Universidad de Aarhus, Dinamarca como profesor asociado en 1979. Durante un año estuvo de licencia de la Universidad de Stanford, pero permaneció en Aarhus después de ese año, dimitiendo de Stanford. Continuó ocupando el puesto en Aarhus hasta 1983, pero en 1982 pasó un año en la Universidad de Pensilvania como profesor asociado visitante, Referencia [5]:

Jorgensen estaba nuevamente en la facultad de la Universidad de Pensilvania a principios de la década de 1980. Esta vez fue también el comienzo de la colaboración con Ola Bratteli que se prolongó durante décadas: muchas publicaciones de investigación conjunta. Otros coautores de estos artículos fueron Derek W Robinson y George A Elliott.

Al final de un año en la Universidad de Pennsylvania, Jorgensen fue nombrado profesor en el Departamento de Matemáticas de la Universidad de Iowa. Renunció a su puesto en Aarhus y se le concedió un año de licencia de Iowa para poder continuar como profesor asociado visitante en la Universidad de Pensilvania y empezar a enseñar en la Universidad de Iowa en 1984. En la entrevista, Referencia [2], ha dicho:

[En la Universidad de Iowa] He estado enseñando la mayoría de los cursos de pregrado, pero en los últimos años principalmente cursos de posgrado, y colaboro con colegas en física (el grupo de teoría) e ingeniería (ingeniería industrial, informática y eléctrica.) Imparto cursos de análisis, análisis funcional, matemáticas aplicadas (especialmente física, procesamiento de señales e imágenes. [Mis principales intereses matemáticos son]) teoría de operadores, análisis funcional, sistemas dinámicos, entre otros. Por lo demás, mis intereses profesionales van desde las matemáticas hasta desde la física, desde la ingeniería (procesamiento de señales e imágenes) hasta las matemáticas financieras (¡ecuaciones diferenciales estocásticas!) ... Disfruto leyendo, también disfruto enseñando, dando seminarios y trabajando con estudiantes, especialmente con mis estudiantes de doctorado.

Jorgensen es autor o coautor de al menos once libros (y ha sido editor de varios más). Damos alguna información sobre los libros de los que es autor o coautor en el final de este artículo biográfico.

Una biografía reciente dice, Referencia 12]:

La lista de publicaciones [de Jorgensen] incluye más de 200 artículos de investigación y 8 libros: en matemáticas, tanto puras como aplicadas (álgebras de operadores y análisis armónico) y física matemática (teoría cuántica). La investigación de Palle Jorgensen cubre un amplio espectro, procesos estocásticos, análisis de ruido blanco, análisis armónico, análisis funcional aplicado, PDE, sistemas dinámicos, algoritmos discretos, análisis estocástico aplicado, y la investigación de Jorgensen hace conexiones con una serie de áreas aplicadas, y ha interactuado con colegas de otros departamentos, tanto de la Universidad de Iowa como de otras universidades de todo el mundo. Esto incluye a colegas en los departamentos de física, ingeniería (departamento de ingeniería informática y eléctrica), departamento de estadística y ciencias actuariales (Universidad de Iowa). Durante los últimos siete años, Jorgensen ha enseñado matemáticas financieras tanto a nivel de posgrado como a nivel de estudios superiores. Curso de pregrado en valores derivados. Estos cursos atraen a estudiantes de matemáticas, estadística y ciencias actuariales, ingeniería, economía y finanzas. Mientras estuvo en la Universidad de Iowa, Jorgensen ha tenido 24 estudiantes de doctorado, con tesis que cubren temas tanto puros como aplicados.

El trabajo editorial que ha realizado Jorgensen incluye: editor de Acta Applicandae Mathematicae; editor de *Proceedings of the American Mathematical Society*; editor de Proyecciones: Revista de Matemática; editor de la Revista Panamericana de Matemáticas; editor de la Revista de Ciencias Matemáticas (nueva serie, Delhi, India); editor del *Journal of Applied Mathematics and Computing,* editor del Journal of Basic & Applied Sciences, editor de Complex Analysis and Operator Theory y editor del Journal of Nonlinear Functional Analysis.

Jorgensen fue elegido miembro de la Academia Danesa de Ciencias Naturales en 1982, año en que se fundó la academia.

Basado en el artículo de JJ O'Connor y EF Robertson
http://www-history.mcs.st-and.ac.uk/Biographies/Jorgensen.html
casanchi.org

Referencias

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14. F H Vasilescu, Review: Operator Commutation Relations, by Robert T Moore and Palle E T Jorgensen (D Reidel, Dordrecht, 1984), *Zentralblatt, European Mathematical Society*. http://homepage.divms.uiowa.edu/~jorgen/Zbl053547020.pdf

Fuentes adicionales

Other pages about Palle Jorgensen:

1. Palle E T Jorgensen's books

Libros de Palle E T Jorgensen:

We have found eleven books authored by Palle E T Jorgensen. There are several texts which Jorgensen has edited but we have omitted them from our list. The information below comes from Prefaces, Publisher's Information, reviews, etc.

Click on a link below to go to the information about that book

Operator commutation relations, Commutation relations for operators, semigroups, and resolvents with applications to mathematical physics and representations of Lie groups (1984) with Robert T Moore

Operators and representation theory. Canonical models for algebras of operators arising in quantum mechanics (1988)

Iterated Function Systems and Permutation Representations of the Cuntz Algebra (1999) with Ola Bratteli

Wavelets through a looking glass. The world of the spectrum (2002) with Ola Bratteli

Representation Theory and Numerical AF-Invariants: The Representations and Centralizers of Certain States on O_{d}*Od* (2004) with Ola Bratteli and Vasyl Ostrovskyi

Analysis and probability: wavelets, signals, fractals (2006)

Iterated Function Systems, Moments, and Transformations of Infinite Matrices (2011) with Keri A Kornelson and Karen L Shuman

Extensions of positive definite functions. Applications and their harmonic analysis (2016) with Steen Pedersen and Feng Tian

Non-commutative analysis (2017) with Feng Tian

Transfer operators, endomorphisms, and measurable partitions (2018) with Sergey Bezuglyi

Harmonic analysis. Smooth and non-smooth (2018)

1. Operator commutation relations, Commutation relations for operators, semigroups, and resolvents with applications to mathematical physics and representations of Lie groups (1984), by Palle E T Jorgensen and Robert T Moore.

1.1. Review by: Derek W Robinson

Mathematical Reviews MR0746138 **(86i:22006)**.

In the last thirty years the theory of continuous one-parameter groups and semigroups on Banach spaces has flourished. This theory, which has its historical roots in the dynamical description of quantum-mechanical systems, has subsequently found applications in many fields of mathematics. The main emphasis of this development has been on the analysis of single groups, or semigroups, and their generators. Less effort has been devoted to the study of the analytic structure of families of noncommuting groups and semigroups. Again such problems found their earliest motivation in quantum theory, notably with Heisenberg's formulation in terms of families of noncommuting observables. Problems of this nature form the principal object of the monograph under review and the authors attempt, with some success, to show that analysis of operator commutation relations leads to a unification of diverse areas of mathematics. ... Some of the topics treated are mass splitting theorems in elementary particle physics, quantum-mechanical commutation relations, integration of Lie algebras, and unitary representations of noncompact groups. Despite this wealth of application the reader obtains the impression that there remains much to discover in commutation theory, and this monograph provides both motivation and a guide to the current state of knowledge.

1.2. Review by: F H Vasilescu.
Zentralblatt MATH Database,
http://homepage.divms.uiowa.edu/~jorgen/Zbl053547020.pdf

Dirac mentioned somewhere that "the noncommutation was really the dominant characteristic of Heisenberg's theory." As is well known, in the Heisenberg formalism of the quantum physics the "observables" are infinite matrices, whereas the Schrödinger formalism uses partial differential operators. These formalisms are the historical roots of two different types of sufficient conditions, which ensure that some formal commutation relation, dealt with in the present book, are mathematically correct. And the very existence of a certain number of formal commutation identities leads to a surprising unification (as well as to a significant range of applications), in spite of the use of two distinct collections of techniques. ... The present work emphasizes the role played by the analysis of infinitesimal and global commutation relations for operators in different areas of mathematics (pure and applied). The authors have chosen the C^{∞}*C*∞-vector approach, as opposed to analytic vectors, because of its suitability to general Banach and locally convex spaces. Many of the results have been obtained by the authors themselves. The book, which is clearly and systematically written, is addressed to all sorts of mathematicians, graduate students and researchers, especially to those interested in the applications of the operator theory to mathematical physics.

2. Operators and representation theory. Canonical models for algebras of operators arising in quantum mechanics (1988), by Palle E T Jorgensen.

2.1. From the Publisher.

Historically, operator theory and representation theory both originated with the advent of quantum mechanics. The interplay between the subjects has been and still is active in a variety of areas. This volume focuses on representations of the universal enveloping algebra, covariant representations in general, and infinite-dimensional Lie algebras in particular. It also provides new applications of recent results on integrability of finite-dimensional Lie algebras. As a central theme, it is shown that a number of recent developments in operator algebras may be handled in a particularly elegant manner by the use of Lie algebras, extensions, and projective representations. In several cases, this Lie algebraic approach to questions in mathematical physics and C*-algebra theory is new; for example, the Lie algebraic treatment of the spectral theory of curved magnetic field Hamiltonians, the treatment of irrational rotation type algebras, and the Virasoro algebra. Also examined are C*-algebraic methods used (in non-traditional ways) in the study of representations of infinite-dimensional Lie algebras and their extensions, and the methods developed by A Connes and M A Rieffel for the study of the Yang-Mills problem. Cutting across traditional separations between fields of specialization, the book addresses a broad audience of graduate students and researchers.

2.2. From the Publisher.

Suitable for advanced undergraduates and graduate students in mathematics and physics, this three-part treatment of operators and representation theory begins with background material on definitions and terminology as well as on operators in Hilbert space. The introductory section concludes with a look at the imprimitivity theorem, which grounds in more mathematical language the work of Wigner on representations of the Poincaré and Galilei groups. The second part of the monograph addresses the algebras of operators in Hilbert space, broadening the mathematics used in earlier versions of quantum theory. There are many examples in which the Hamiltonian, the operator that translates a quantum system in time, can be written as a polynomial in elements of an underlying Lie algebra. This section deals with properties of such operators. Part 3 explores covariant representation and connections, with a particular focus on infinite-dimensional Lie algebras. Connections to mathematical physics are stressed throughout the text, which concludes with three helpful appendixes, including a Guide to the Literature.

2.3. From the Publisher.

Algebras of operators arise frequently in the study of representations of Lie groups, both finite-dimensional and infinite-dimensional. This book begins with extensive background material that covers definitions and terminology, operators in Hilbert space, and the imprimitivity theorem. Advancing to considerations of the algebras of operators in Hilbert space, the heart of the text examines domains of representations, operators in the enveloping algebra, and spectral theory. The final section explores covariant representations and connections, with a particular focus on infinite-dimensional Lie algebras. A helpful Appendix on the integrability of Lie algebras concludes the text.

2.4. Review by: Konrad Schmüdgen.
Mathematical Reviews MR0919948 **(89e:47001)**.

The book under review is about algebras of unbounded operators in Hilbert space which arise in the representation theory of (both finite-dimensional and infinite-dimensional) Lie algebras or more precisely of the corresponding universal enveloping algebras. It consists of three parts. Part I collects some background material such as basic concepts on Hilbert space operators and the imprimitivity theorem. Part II contains a number of results on representations of finite-dimensional Lie algebras which appear for the first time in a book. ... Part III is concerned with representations of some infinite-dimensional Lie algebras which are important in mathematical physics and in operator algebra theory. ... This book is a research monograph. Many results presented therein have been obtained by the author and his co-workers such as O Bratelli, J Cuntz, G A Elliott, D E Evans, F M Goodman, W H Klink, R T Moore and D W Robinson. Often results from the literature are used in the text and additional results are discussed.

3. Iterated Function Systems and Permutation Representations of the Cuntz Algebra (1999), by Ola Bratteli and Palle E T Jorgensen.

3.1. From the Publisher.

This book is intended for graduate students and research mathematicians working in functional analysis.

3.2. Review by: Paul Jolissaint.
Mathematical Reviews MR1469149 **(99k:46094a)**.

The main theme ... is the study of some representations of the Cuntz algebra O_{N}*ON*, coming from suitable dynamical systems .. [It] is mainly devoted to the study of general permutative representations of O_{N}*ON*; it contains the construction of a universal permutative (nonseparable) representation ...

4. Wavelets through a looking glass. The world of the spectrum (2002), by Ola Bratteli and Palle E T Jorgensen.

4.1. From the Publisher.

This book combining wavelets and the world of the spectrum focuses on recent developments in wavelet theory, emphasizing fundamental and relatively timeless techniques that have a geometric and spectral-theoretic flavour. The exposition is clearly motivated and unfolds systematically, aided by numerous graphics. This self-contained book deals with important applications to signal processing, communications engineering, computer graphics algorithms, qubit algorithms and chaos theory, and is aimed at a broad readership of graduate students, practitioners, and researchers in applied mathematics and engineering. The book is also useful for other mathematicians with an interest in the interface between mathematics and communication theory.

4.2. Review by: Gilbert Walter.
Mathematical Reviews MR1913212 **(2003i:42001)**.

In the last few years, a plethora of books on wavelets have appeared. Most have been variations on the same themes which were covered in some detail by Daubechies in 1992 and by Meyer in 1990. So I was expecting more of the same in this book. But it's different, a lot different. One difference is its point of view which derives more from mathematical physics than from signal processing. This enables the authors to take a fresh look at the subject and develop a new intuition for many topics. It particular, they make more extensive use of spectral theory than is usual in the subject. The book also has a number of unusual aspects in its organization. Each of the six chapters begins with a relatively intuitive vignette which is written in a more leisurely style than other parts of the book. These sections give the book much of its flavour; they are entitled respectively: "Overture: why wavelets", "The dangers of navigating the landscape of wavelets", "The world of the spectrum", "A slanted matrix from dynamics", "The fine structure of correlation", and "The other side of wavelets". Each gives an introduction to the topics in the chapter and explains why they are discussed and where they come from. There are connections with many other areas of mathematics and physics not usually associated with wavelet theory.

4.3. Review by: Judith A Packer.
SIAM Review **46** (2) (2004), 368-372.

Mere words cannot adequately describe all the great features of the new book by

Ola Bratteli and Palle Jorgensen, which has something for everyone of all mathematical persuasions. Whatever your feelings about this book, you will be left breath less by its scope. Subband filters, qubits from physics, loop groups, homotopy theory for wavelets and related index theorems, Cuntz C*-algebras, transfer operators and a Perron-Frobenius theory for their eigenvalues, cycles from dynamics, isospectral approximation - all these are discussed in addition to the standard theory of wavelets and multiresolution analysis which can be found in the books by Y Meyer, I Daubechies, or E Hernández and G Weiss, to name just a few. ... it is clear that the authors view wavelets almost as living organisms, and one of their aims is to "show that wavelets have a life of their own outside the Fourier Kingdom: that is the world of cascades, algorithms, and the spectrum." Their love of the subject is evident from their writing, and is contagious to the reader. ... In conclusion, I would say to everyone that this is a fun book, full of exciting new results, written by two world-renowned experts in the field, which makes connections between a variety of important areas in pure and applied mathematics. The authors are to be commended for making these connections. It is certainly worth having on your "wish list" of books to read this year.

4.4. Review by: Ole Christensen.
Zentralblatt MATH Database,
http://homepage.divms.uiowa.edu/~jorgen/quick2.pdf

This is a quite unusual wavelet book, with a fresh view on the subject. While the vast majority of wavelet books concentrate on multiresolution analysis and its applications, this book treats the topic from an operator theoretic point of view, with focus on techniques having a geometric or spectral theoretic flavour. In fact, while parts of the book could be used in a wavelet course, other parts would be suitable in a course directed towards operator theory. Each chapter (and some of the sections) starts with an informal tutorial, which explains the main ideas in less technical terms. Furthermore chapter 1 contains a section with terminology, which explains the main concepts and the meaning of the key words in e.g., engineering, physics, and mathematics. Both features certainly helps the reader. Each chapter concludes with exercises of varying difficulty.

5. Representation Theory and Numerical AF-Invariants: The Representations and Centralizers of Certain States on O_{d}*Od* **(2004), by Ola Bratteli, Palle E T Jorgensen and Vasyl Ostrovskyi.**

5.1. From the Introduction.

The origin of this monograph was the desire to understand certain concrete operator relations arising as realizations of filtering processes in signal theory. Our research did, however, lead us naturally in the direction of analyzing certain non-commutative dynamical systems and their fixed points and states. In this introduction, we give an overview of the contents of the monograph in three stages: First (1) a very general discussion, then (2) a discussion with more specific definitions and details, and finally (3) a detailed technical account of what the paper actually contains.

5.2. General discussion and motivation.

This monograph is centred around the issue of distinguishing a particular family of

AF-algebras, those which arise as the centralizers of certain states on Cuntz algebras - or equivalently, as the fixed-point algebras under certain one-parameter subgroups of the gauge action. (The Cuntz algebras are the range of a functor from Hilbert space into C*-algebras. The term AF-algebra is short for approximately finite-dimensional C*-algebra. Both the Cuntz algebras and the AF-algebras play a role in several areas of mathematics, e.g., K-theory and dynamical systems, and in applications, for example to statistical mechanics.) While AF-algebras generally are classified up to stable isomorphism by the equivalence classes of their Bratteli diagrams, or by the isomorphism classes of their ordered dimension groups, none of the three items in this triple of equivalence classes, of AF-algebras, Bratteli diagrams, or ordered dimension groups, respectively, is especially amenable to computation. In this memoir, we therefore try to approach the subject via classes of concrete examples, which do in fact admit algorithms for distinguishing isomorphism classes, in particular by the computation of numerical invariants which distinguish these classes. The examples are chosen so they illustrate in a concrete manner the main issues of computation in each of the three incarnations, AF-algebras, Bratteli diagrams, or dimension groups.

5.3. Review by: Andrew James Dean.

Mathematical Reviews MR2030387 **(2005i:46069)**.

This memoir opens with a discussion of the representations of the Cuntz algebras O_{n}*On*. The main result of this introductory portion is that any non-degenerate representation of O_{n}*On* on a separable Hilbert space may be expressed in a natural way as a direct integral over the spectrum of a canonical maximal abelian subalgebra of O_{n}*On*. The main part of the paper is concerned with distinguishing the isomorphism classes of the fixed point subalgebras of O_{n}*On* under certain actions of \mathbb{R}R.

6. Analysis and probability: wavelets, signals, fractals (2006), by Palle E T Jorgensen.

6.1. From the Preface.

If people do not believe that mathematics is simple, it is only because they do not realize how complicated life is. -- John von Neumann

While this is a course in analysis, our approach departs from the beaten path in some ways. Firstly, we emphasize a variety of connections to themes from neighbouring fields, such as wavelets, fractals and signals; topics typically not included in a graduate analysis course. This in turn entails excursions into domains with a probabilistic flavour. Yet the diverse parts of the book follow a common underlying thread, and together they constitute a good blend; each part in the mix naturally complements the other. In fact, there are now good reasons for taking a wider view of analysis, for ex ample the fact that several applied trends have come to interact in new and exciting ways with traditional mathematical analysis - as it was taught in graduate classes for generations. One consequence of these impulses from "outside" is that conventional boundaries between core disciplines in mathematics have become more blurred. Fortunately this branching out does not mean that students will need to start out with any different or additional prerequisites. In fact, the ideas involved in this book are intuitive, natural, many of them visual, and geometric. The required background is quite minimal and it does not go beyond what is typically required in most graduate programs.

We believe that now is a good time to slightly widen the horizons of the subject "analysis" as we teach it by stressing its relations to neighbouring fields; in fact we believe that analysis is thereby enriched. Despite the inclusion of themes from probability and even from engineering, the course still has an underlying core theme: A constructive approach to building bases in function spaces. The word "constructive" here refers to our use of recursive algorithms. As it turns out, the algorithmic ideas involved are commonly used in such diverse areas as wavelets, fractals, signal and image processing. And yet they share an underlying analysis core which we hope to bring to light. Our inclusion here of some applied topics (bordering probability theory and engineering) we believe is not only useful in itself, but more importantly, core mathematics, and analysis in particular have benefited from their many interconnections to trends and influences from the "outside" world. Yet our wider view of the topic analysis only entails a minor adjustment in course planning. Our branching out to some applications will be guided tours: to topics from probability theory (e.g., to certain random-walk models), and to signal and image processing. The ideas are presented from scratch, are easy to follow, and they do not require prior knowledge of probability or of engineering. But we will go a little beyond the more traditional dose of measure theory and matrix algebra that is otherwise standard or conventional fare in most first-year graduate courses. For those reasons we believe the book may also be suitable for a "second analysis course," and that it leaves the instructor a variety of good options for covering a selection of neighbouring disciplines and applications in more depth.

6.2. Review by: Judith A Packer.
Mathematical Reviews MR2254502 **(2008a:42030)**.

In this book, an entry in Springer's Graduate Texts in Mathematics series, the author has made the effort to put enough material on probability theory, wavelets and frames, operators, Cuntz algebras, and the Perron-Frobenius theorem and its relationship to the Ruelle operator so that a reader without much background in any of these topics can prepare to read the more detailed research articles of the author and his collaborators .... He has also made an admirable attempt to bridge the "language divide" between theoretical mathematicians and engineers, by explaining in a very understandable fashion to each side many of the diagrams and terms that one side might consider as jargon on the part of the other. Also, many of his explanations, particularly those involving probability theory, have not appeared together in book form prior to this latest effort by the author. The book certainly succeeds in showing a great number of connections between a variety of areas in pure mathematics, applied mathematics, and engineering. ... this is a useful book for those who want to understand the important research done by the author and his many collaborators in recent years, especially for those who might not have the appropriate background either in mathematics or in engineering and signal processing. At the end of every chapter and appendix, there are exercises that point the readers both backward, towards the direction of elementary results from operator theory and Hilbert space theory, and forward, with an eye towards the author's latest research results. The illustrations, including graphs of wavelets and signal processing diagrams, are plentiful and illuminating. The multitude of topics in mathematics covered and the links between them that are described in this book mean that there is something for everyone.

7. Iterated Function Systems, Moments, and Transformations of Infinite Matrices (2011), by Palle E T Jorgensen, Keri A Kornelson and Karen L Shuman.

7.1. From the Publisher.

The authors study the moments of equilibrium measures for iterated function systems (IFSs) and draw connections to operator theory. Their main object of study is the infinite matrix which encodes all the moment data of a Borel measure on \mathbb{R}^{d}R*d* or \mathbb{C}C. To encode the salient features of a given IFS into precise moment data, they establish an interdependence between IFS equilibrium measures, the encoding of the sequence of moments of these measures into operators, and a new correspondence between the IFS moments and this family of operators in Hilbert space. For a given IFS, the authors' aim is to establish a functorial correspondence in such a way that the geometric transformations of the IFS turn into transformations of moment matrices, or rather transformations of the operators that are associated with them.

7.2. Review by: Peter R Massopust.
MathematicalReviews MR2858536 **(2012g:47002)**.

This memoir is devoted to the study of moments of equilibrium measures for iterated function systems (IFSs) and their connections to operator theory. The focus of this study is the infinite matrix of all the moment data of a Borel measure on \mathbb{R}^{d}R*d*, d*d* in \mathbb{N}N, or \mathbb{C}C. In order to encode the relevant features of a given IFS into precise moment data, the authors establish a functorial correspondence between the geometric transformations of the IFS and the transformations of moment matrices. The latter are associated with a family of operators on Hilbert space. ... This is a well-written and very interesting memoir article that strives to connect different aspects of the moment problem and to elucidate the relationship between moment matrices, IFSs, operators and spectral properties. It is highly recommended for researchers interested in this subject.

8. Extensions of positive definite functions. Applications and their harmonic analysis (2016), by Palle E T Jorgensen, Steen Pedersen and Feng Tian.

8.1. From the Publisher.

This monograph deals with the mathematics of extending given partial data-sets obtained from experiments; Experimentalists frequently gather spectral data when the observed data is limited, e.g., by the precision of instruments; or by other limiting external factors. Here the limited information is a restriction, and the extensions take the form of full positive definite function on some prescribed group. It is therefore both an art and a science to produce solid conclusions from restricted or limited data. While the theory of is important in many areas of pure and applied mathematics, it is difficult for students and for the novice to the field, to find accessible presentations which cover all relevant points of view, as well as stressing common ideas and interconnections. We have aimed at filling this gap, and we have stressed hands-on-examples.

8.2. Review by: Douglas R Farenick.
Mathematical Reviews MR3559001.

This book addresses the problem of extending positive definite functions defined on subsets S of a locally compact group G to positive definite functions on G. ... The authors provide a quite readable account of the positive definite extension problem. Among the topics that are covered are Pólya extensions, Mercer operators, Green's functions, spectral theory and models, and comparing the reproducing kernel Hilbert spaces for various pairs of locally-defined positive definite functions. The authors conclude with a discussion of some of the book's achievements and related open problems. The authors' decision to emphasize concrete examples enhances the book's value and interest, as do the efforts to make connections to other subjects such as stochastic processes and Brownian motion. At the same, several important concepts are treated at a sufficiently general and abstract level, making this book a welcome addition to the literature.

9. Non-commutative analysis (2017), by Palle E T Jorgensen and Feng Tian.

9.1. From the Publisher.

The book features new directions in analysis, with an emphasis on Hilbert space, mathematical physics, and stochastic processes. We interpret "non-commutative analysis" broadly to include representations of non-abelian groups, and non-abelian algebras; emphasis on Lie groups and operator algebras (C^*-algebras and von Neumann algebras.) A second theme is commutative and non-commutative harmonic analysis, spectral theory, operator theory and their applications. The list of topics includes shift invariant spaces, group action in differential geometry, and frame theory (over-complete bases) and their applications to engineering (signal processing and multiplexing), projective multi-resolutions, and free probability algebras. The book serves as an accessible introduction, offering a timeless presentation, attractive and accessible to students, both in mathematics and in neighbouring fields.

9.2. Preface by W Polyzou.

Progress in science, engineering and mathematics comes fast and it often requires a significant effort to keep up with the advances in other fields that impact applications. Functional analysis (especially operators in Hilbert space, unitary representations of Lie groups, and spectral theory) is one discipline that impacts my physics research. Bringing my students up to speed with the subject facilitates their ability to efficiently perform research, however the typical curriculum in functional analysis courses is not directed to practitioners whose primary objective is applications. This is also reflected in rho many excellent available texts on the subject , which primarily focus on the mathematics, and arc directed at students aspiring to a career in mathematics. I have been fortunate to have Palle Jorgensen as a colleague. He participates in a weekly joint mathematical physics seminar that is attended by faculty and students from both departments. It provides a forum to address questions related to the role of mathematics in physics research. Professor Jorgensen has a healthy appreciation of applications of functional analysis; in these seminars he has been at the centre of discussions on a diverse range of applications involving wavelets, reflection positivity, path integrals, entanglement, financial mathematics, and algebraic field theory. A number of the mathematically inclined students in my department have benefited from taking the functional analysis course taught by Professor Jorgensen. These students are motivated to enrol in his class because the course material includes a significant discussion of applications of functional analysis to subjects that interest them. This book is based on the course that Professor Jorgensen teaches on functional analysis. It fills in a gap that is not addressed by the many excellent available texts on functional analysis, by using applications to motivate basic results in functional analysis. The way that it uses applications makes the material more accessible to students; particularly for students who will eventually find careers in related disciplines. The book also points to additional reference material for students who are motivated to learn more about a specific topic.

9.3. From the Preface.

There are already many books in Functional Analysis, so why another?
The main reason is that we feel there is a need: in the teaching at the beginning graduate level; more flexibility, more options for students and instructors in pursuing new directions. And aiming for a book which will help students with primary interests elsewhere to acquire a facility with tools of a functional analytic flavour, say in spectral theory for operators in Hilbert, space, in commutative and non-commutative harmonic analysis, in PDE, in numerical analysis, in stochastic processes, or in physics.

The book. Over the decades, Functional Analysis, and the theory of operators in Hilbert space, have been enriched and inspired on account of demands from neighbourlng fields, within mathematics, harmonic analysis (wavelets and signal processing), numerical analysis (finite element methods, discretization), PDEs (diffusion equations, scattering theory), representation theory; iterated function systems (fractals, Julia sets, chaotic dynamical systems), ergodic theory, operator algebras, and many more. And neighbouring areas, probability/statistics (for example stochastic processes, Ito and Malliavin calculus), physics (representation of Lie groups, quantum field theory), and spectral theory for Schrödinger operators. The book is based on a course sequence (two-semesters) taught, over the years at the University of Iowa, by the first-named author. The students in the course made up a mix: some advanced undergraduates, but most of them, first or second year graduate students (from mathenmatics, as well as some from physics and statistics.) We have subsequently expanded the course notes taken by the second-named author: we completed several topics from the original notes, and we added a number of others, so that the book is now self-contained, and it covers a unified theme; and yet it stresses a multitude of applications. And it offers flexibility for users.

9.4. Review by: Rolf Gohm.
MathematicalReviews MR3642406.

This book is an interesting attempt to give a course about Functional Analysis which avoids some of the lengthy technicalities and abstractions of more traditional texts in favour of a more direct working platform with more direct connections to applications. Such an approach is possible because from original motivations in quantum theory and noncommutativity, leading to an emphasis on Hilbert space and spectral theory, we arrive at a core body of knowledge with in fact many roads to many old and new applications. ... It is inspiring and original how old material (for example the work of von Neumann about unbounded operators) is combined and mixed with new material (for example operators defined from infinite graphs). There is always something unexpected included in each chapter, which one is thankful to see explained in this context and not only in research papers which are more difficult to access. In the opinion of the reviewer some chapters are better than others: some are fully enjoyable, while in others it is not easy to find the thread through the wealth of material, and also sometimes the authors' approach doesn't seem to work as hoped and the more advanced parts are not really accessible without consulting the references. Doing the latter is of course always an option and luring readers into many interesting directions is definitely a strength of this book.

10. Transfer operators, endomorphisms, and measurable partitions (2018), by Sergey Bezuglyi and Palle E T Jorgensen.

10.1. From the Preface.

The subject of this book stands at the crossroads of ergodic theory and measurable dynamics. With an emphasis on irreversible systems, the text presents a framework of multi-resolutions tailored for the study of endomorphisms, beginning with a systematic look at the latter. This entails a whole new set of tools, often quite different from those used for the "easier" and well-documented case of automorphisms. Among them is the construction of a family of positive operators (transfer operators), arising naturally as a dual picture to that of endomorphisms. The setting (close to one initiated by S Karlin in the context of stochastic processes) is motivated by a number of recent applications, including wavelets, multi-resolution analyses, dissipative dynamical systems, and quantum theory. The automorphism-endomorphism relationship has parallels in operator theory, where the distinction is between unitary operators in Hilbert space and more general classes of operators such as contractions. There is also a non-commutative version: While the study of automorphisms of von Neumann algebras dates back to von Neumann, the systematic study of their endomorphisms is more recent; together with the results in the main text, the book includes a review of recent related research papers, some by the co-authors and their collaborators.

10.2. From the Introduction.

While the mathematical structures of positive operators, endomorphisms, transfer operators, measurable partitions, and Markov processes arise in a host of settings, both pure and applied, we propose here a unified study. This is the general setting of dynamics in standard Borel and measure spaces. Hence the corresponding linear structures are infinite-dimensional. Nonetheless, we prove a number of analogues of the more familiar finite-dimensional settings, for example, the Perron-Frobenius theorem for positive matrices, and the corresponding Markov chains.

11. Harmonic analysis. Smooth and non-smooth (2018), by Palle E T Jorgensen.

11.1. From the Publisher.

There is a recent and increasing interest in harmonic analysis of non-smooth geometries. Real-world examples where these types of geometry appear include large computer networks, relationships in datasets, and fractal structures such as those found in crystalline substances, light scattering, and other natural phenomena where dynamical systems are present. Notions of harmonic analysis focus on transforms and expansions and involve dual variables. In this book on smooth and non-smooth harmonic analysis, the notion of dual variables will be adapted to fractals. In addition to harmonic analysis via Fourier duality, the author also covers multi-resolution wavelet approaches as well as a third tool, namely, L^2 spaces derived from appropriate Gaussian processes. The book is based on a series of ten lectures delivered in June 2018 at a Conference Board of the Mathematical Sciences (CBMS) conference held at Iowa State University.

11.2. Preview.

Smooth harmonic analysis refers to harmonic analysis over a connected or locally connected domain - typically Euclidean space or locally connected subsets of Euclidean space. The classical example of this is the existence of Fourier series expansions for square integrable functions on the unit interval. Non-smooth harmonic analysis then refers to harmonic analysis on discrete or disconnected domains - typical examples of this setting are Cantor like subsets of the real line and analogous fractals in higher dimensions. In 1998, Jorgensen and Steen Pedersen proved a remarkable result: there exists a Cantor like set (of Hausdorff dimension 1/2) with the property that the uniform measure supported on that set is spectral, meaning that there exists a sequence of frequencies for which the exponentials form an orthonormal basis in the Hilbert space of square integrable functions with respect to that measure. This surprising result, together with results of Robert Strichartz, has lead to a plethora of new research directions in non-smooth harmonic analysis.

Research that has been inspired by this surprising result includes: fractal Fourier analyses (fractals in the large), spectral theory of Ruelle operators; representation theory of Cuntz algebras; convergence of the cascade algorithm in wavelet theory; reproducing kernels and their boundary representations; Bernoulli convolutions and Markov processes. The remarkable aspect of these broad connections is that they often straddle both the smooth and non-smooth domains. This is particularly evident in Jorgensen's research on the cascade algorithm, as wavelets already possess a "dual" existence in the continuous and discrete worlds, and also his research on the boundary representations of reproducing kernels, as the non-smooth domains appear as boundaries of smooth domains. In work with Dorin Dutkay, Jorgensen showed that the general affine IFS-systems, even if not amenable to Fourier analysis, in fact do admit wavelet bases, and so in particular can be analysed with the use of multi-resolutions. In recent work with Herr and Weber, Jorgensen has shown that fractals that are not spectral (and so do not admit an orthogonal Fourier analysis) still admits a harmonic analysis as boundary values for certain subspaces of the Hardy space of the disc and the corresponding reproducing kernels within them.

The lectures to be given by Jorgensen will cover the following overarching themes: the harmonic analysis of Cantor spaces (and measures) arising as fractals (including fractal dust) and iterated function systems (IFSs), as well as the methods used to study their harmonic analyses that span both the smooth and non-smooth domains. A consequence of the fact that these methods form a bridge between the smooth and non-smooth domain is that the topics to be discussed - while on the surface seem largely unrelated - actually are closely related and together form a tightly focused theme. The breadth of topics will attract a broader audience of established researchers, while the interconnectedness and sharply focused nature of these topics will prove beneficial to beginning researchers in non-smooth harmonic analysis.

11.3. Review by: Javier Duoandikoetxea.

Mathematical Reviews MR3838440.

This book is a presentation of several aspects of harmonic analysis on non-smooth domains, beyond the classical theory, which is termed smooth by the author. It is an interesting topic because the usual tools of Fourier analysis are not always available when working in the non-smooth setting and different points of view have been developed in the last twenty years to overcome the difficulties. Palle E T Jorgensen is one of the major contributors to the field, and his set of lectures at the CBMS conference that is at the origin of this book is to a large extent a description of a variety of his own work. ... In summary, the book covers various aspects of a recently developed theory of non-smooth harmonic analysis as seen by one of the main experts in the field, who has devoted most of his extensive work to the subject. Through different techniques ranging from Fourier analysis to operator theory, stochastic processes and more, the reader will find a very interesting guide to the field.

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